The $\eta_c\gamma\gamma^*$ transition form factor in the decay $\eta_c \to \gamma\ell^+\ell^-$ and in the crossed channels $\gamma e^- \to \eta_c e^-$ and $e^+e^- \to \eta_c\gamma$

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Abstract. The Dalitz distribution and the effective mass spectrum of the lepton pair produced in the decay $\eta_c \to \gamma\ell^+\ell^-$, as well as the differential cross sections for the crossed channels $\gamma e^- \to \eta_c e^-$ and $e^+e^- \to \eta_c\gamma$, are calculated in terms of the sole electromagnetic transition form factor for the vertex $\eta_c \to \gamma\gamma^*$, where γ^* is a virtual photon with a space-like (for $\gamma e^- \to \eta_c e^-$) or a time-like (for $e^+e^- \to \eta_c\gamma$ and $\eta_c \to \gamma\ell^+\ell^-$) four-momentum. Explicit parameterizations of the transition form factor are found in two different approaches, namely in the vector dominance model and in a QCD-inspired consideration on the basis of the process $c\bar{c} \to \gamma\ell^+\ell^-$, with zero relative three-momentum of the annihilating quarks.

1 Introduction

The radiative decays of mesons and baryons are an interesting source of important information for hadron spectroscopy and hadron electrodynamics [1,2]. For example, it is worth recalling the role of such decays in determining the charmonium spectrum [2]. Among radiative decays, especially intriguing are those where lepton pairs are produced. A classical example is the process $\pi^0 \to \gamma e^+e^-$ [3], which is sensitive to the radius of the corresponding form factor. Such a decay is typical of any neutral pseudoscalar meson P ($= \eta, \eta', \eta_c$, etc.):

$$P \to \gamma\ell^+\ell^-, \quad \ell = e, \mu.$$

The corresponding transition electromagnetic form factors of $P \to \gamma\gamma^*$ decay are complex functions of t, the four-momentum transfer squared, in the time-like region $4m_\ell^2 < t \le m^2$, where m (m_ℓ) is the mass of P (ℓ). In particular the decay $\eta_c \to \gamma\ell^+\ell^-$ offers the possibility of studying the transition form factor in a relatively large region of time-like momentum transfer,

$$4m_\ell^2 \le t \le 9\,\mathrm{GeV}^2.$$

A strong theoretical interest in the transition form factors of pseudoscalar mesons was stimulated by the experimental results of the CLEO collaboration [4,5], which measured these form factors at space-like momentum transfers, using the well known 2γ exchange mechanism in the
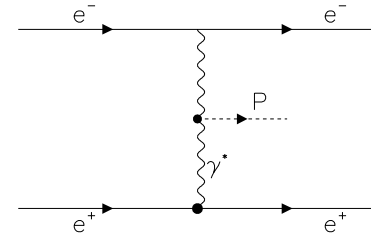

Fig. 1. 2γ mechanism for the $e^+e^- \to e^+e^-P$ process

reaction $e^+e^- \to e^+e^-P$ (Fig. 1). If one lepton, say the positron, is not detected, we can study the process of photoproduction of pseudoscalar mesons on electrons, $\gamma e^- \to Pe^-$, in an unusual kinematics of colliding particles, γ and e^-. In the case of π^0 production the corresponding form factor was measured up to $t = -8\,\mathrm{GeV}^2$, where, in principle, perturbative QCD applies [6–9]. Recently also the $\eta_c\gamma$ transition form factor was determined [10] for two different values of t in the space-like region.

A special interest has the decay $\eta_c \to \gamma\ell^+\ell^-$, as a generalization of the decay $\eta_c \to \gamma\gamma$, whose branching ratio is known [11,10] (see also predictions by dispersive methods [12–16]). The availability of a form factor instead of a single constant, as in the case of $\eta_c \to 2\gamma$ decay, allows a better insight of the structure of the η_c-meson and of perturbative QCD. It is also interesting to look at the predictions of the "old" approach of vector dominance model (VDM) in the unusual region of charmed particle decays [17].

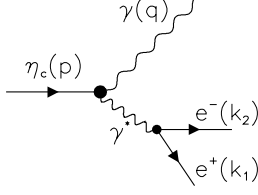This paper is devoted to a global approach to the study of all possible crossed channels, i.e.,

Fig. 2. Diagram for the process $\eta_c \to \gamma e^+ e^-$, with notation of particle four-momenta

$$\eta_c \to \gamma\ell^+\ell^-, \quad \gamma e^- \to \eta_c e^-, \quad e^+ e^- \to \eta_c \gamma,$$

whose characteristics are driven by a single transition form factor. The numerical estimates are performed both within VDM and QCD, considering, in the latter case, the process $c\bar{c} \to \gamma\ell^+\ell^-$ as the basic mechanism generating the $\eta_c \to \gamma\gamma*$ transition form factor.

Section 2 is dedicated to the Dalitz plot distribution of the decay $\eta_c \to \gamma\ell^+\ell^-$. In Sect. 3 we propose two models for calculating the transition form factor $\eta_c \to \gamma\gamma*$, one based on VDM and the other on QCD. In Sect. 4 we study the reaction $e^+ e^- \to \eta_c \gamma$. Section 5 is dedicated to the process $\gamma e^- \to \eta_c e^-$. Finally, in Sect. 6 we draw some conclusions.

2 The decay $\eta_c \to \gamma\ell^+\ell^-$

We start by considering the more general case of the decay of a meson M into a real γ and a lepton pair. The differential decay width reads

$$d\Gamma = (2\pi)^4 \frac{\overline{|\mathcal{M}|^2}}{2m} \delta^4(p - q - k_1 - k_2)$$
$$\times \frac{d^3q}{(2\pi)^3 2\omega} \frac{d^3k_1}{(2\pi)^3 2E_1} \frac{d^3k_2}{(2\pi)^3 2E_2}, \quad (1)$$

where ω, E_1 and E_2 are the energies of the final γ, e^+ and e^-, respectively.

The notation of the four-momenta is shown in Fig. 2; the line above the modulus squared of the matrix element $|\mathcal{M}|^2$ denotes the sum over the polarizations of all the particles produced in the decay.

The standard form of the matrix element $\mathcal{M}$ for the $M \to \gamma\ell^+\ell^-$ decay in the one-photon approximation is

$$\mathcal{M} = \frac{e^2}{t} \ell_\mu \mathcal{J}^\mu,$$

where $t = (k_1 + k_2)^2$ is the effective mass squared of the leptonic pair, $\ell_\mu = \bar{u}(k_2)\gamma_\mu u(-k_1)$ and $\mathcal{J}_\mu$ is the electromagnetic current in the decay $M \to \gamma\gamma*$ [1]. According to the usual notation we obtain

$$\overline{|\mathcal{M}|^2} = \frac{e^4}{t^2} L_{\mu\nu} W^{\mu\nu}, \quad (2)$$

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[1] In the decay considered here, $\mathcal{J}_\mu$ is proportional to the electric charge e

where

$$L_{\mu\nu} = \overline{\ell_\mu \ell_\nu^*}, \quad W^{\mu\nu} = \overline{\mathcal{J}^\mu \mathcal{J}^{\nu*}}. \quad (3)$$

Substituting (2) into (1), and integrating over all variables except $E_{1,2}$, we get the two-lepton Dalitz distribution:

$$d\Gamma = \frac{\alpha^2}{4\pi} \frac{L_{\mu\nu} W^{\mu\nu}}{t^2 m} dE_1 dE_2,$$

where $\alpha = e^2/4\pi$. However, for our aims it is more convenient to express E_1 and E_2 in terms of the dimensionless variables $x = t/m^2$ and $y = 2(E_1 - E_2)/m$ [18], which are defined in the ranges $\delta \le x \le 1$ and $-y_0 \le y \le y_0$, $y_0 = (1 - x)(1 - \delta/x)^{1/2}$ and $\delta = 4m_\ell^2/m^2$. Furthermore, we take into account, wherever possible, symmetry properties:

– Gauge invariance – $k \cdot \ell = k \cdot \mathcal{J} = 0$ – implies that the product $L_{\mu\nu} W^{\mu\nu}$ can be rewritten in terms of the space components of the two tensors above, i.e.,

$$L_{\mu\nu} W^{\mu\nu} = (L_{xx} + L_{yy})W_{xx} + \frac{t^2}{k_0^4} L_{zz} W_{zz}, \quad (4)$$

where k_0 denotes the energy of the γ^*. Here we have adopted a reference frame in which the meson M is at rest and the z-axis is parallel to the momentum $\vec{k}$ of the virtual photon.

– Parity invariance allows one to write

$$W_{ij} = (\delta_{ij} - \hat{k}_i \hat{k}_j)W_1(t) + \hat{k}_i \hat{k}_j W_2(t), \quad \hat{\vec{k}} = \vec{k}/|\vec{k}|,$$

where $W_{1(2)}(t)$ is the real structure function (SF) which describes the production of γ^* with transverse (longitudinal) polarization.

Taking all that into account, calculations yield

$$d\Gamma = \frac{\alpha^2}{16\pi} W_1(t) \frac{dx dy}{xm} \left[1 + \frac{4m_\ell^2}{t} + R_L(t)\frac{t}{k_0^2}\right.$$
$$\left. + y^2 \frac{m^2}{4\vec{k}^2} \left(1 - R_L(t)\frac{t}{k_0^2}\right)\right],$$

$$R_L(t) = W_2(t)/W_1(t), \quad k_0 = \frac{m^2 + t}{2m}, \quad \vec{k}^2 = \frac{(m^2 - t)^2}{4m^2}. \quad (5)$$

One can see that the differential width, which characterizes the Dalitz distribution for the decay $M \to \gamma\ell^+\ell^-$, is symmetric with respect to the exchange $E_1 \leftrightarrow E_2$, in agreement with the C invariance of the electromagnetic interaction.

The specific dependence of $d^2\Gamma/dx dy$ on the variable y,

$$d^2\Gamma/dx dy = a(x) + y^2 b(x),$$

is a direct consequence of one-photon mechanism. It has the same origin as the $\cot^2(\theta_e/2)$ dependence of the differential cross section in electron–hadron scattering (where θ_e is the electron scattering angle in the laboratory system) and as the $\cos^2\theta$ dependence of the differential cross section for the inclusive process $e^+ e^- \to h\, X$, where h denotes the final detected hadron and θ is the angle between

the three-momenta of the hadron and of the electron in the overall CMS.

One can see that the value of $W_1(0)$ determines the probability of the radiative decay width $\Gamma_0 \equiv \Gamma(M \to 2\gamma)$, i.e.,

$$\Gamma_0 = \frac{1}{2}(2\pi)^4 e^2 \frac{W_{xx} + W_{yy}}{2m} \int \delta^4(p-q-k)$$
$$\times \frac{\mathrm{d}^3 k}{(2\pi)^3 2\omega_1} \frac{\mathrm{d}^3 q}{(2\pi)^3 2\omega_2} = \frac{\alpha}{4m} W_1(0).$$

This allows one to rewrite the Dalitz distribution in the following form:

$$\frac{1}{\Gamma_0} \frac{\mathrm{d}^2\Gamma}{\mathrm{d}x\mathrm{d}y}(M \to \gamma\ell^+\ell^-) = \frac{\alpha}{4\pi} R_T(t) \frac{1}{x} \left[1 + \frac{4m_\ell^2}{t}\right.$$
$$\left. + R_L(t)\frac{t}{k_0^2} + \frac{y^2}{4\vec{k}^2}m^2 \left(1 - R_L(t)\frac{t}{k_0^2}\right)\right], \quad (6)$$

where $R_T(t) = W_1(t)/W_1(0)$ and $R_L(t)$ is defined in (5).

Therefore the study of the energy distribution of leptons in $M \to \gamma\ell^+\ell^-$ in the Dalitz plane allows one to determine $R_L(t)$ and $R_T(t)$, i.e., the two fundamental quantities of the decay considered, as functions of the effective mass of the $\ell^+\ell^-$ system.

The result of the integration in (6) over the variable y determines the $\ell^+\ell^-$ effective mass spectrum in the decay $M \to \gamma\ell^+\ell^-$ [19]:

$$\frac{1}{\Gamma_0} \frac{\mathrm{d}\Gamma}{\mathrm{d}x}(M \to \gamma\ell^+\ell^-) = \frac{2\alpha}{3\pi} R_T(t) \frac{(1-x)}{x} \left(1 + 2\frac{m_\ell^2}{t}\right)$$
$$\times \sqrt{1 - \frac{4m_\ell^2}{t}} \left[1 + R_L(t)\frac{t}{2k_0^2}\right]. \quad (7)$$

Turning to the specific decay $\eta_c \to \gamma\gamma^*$, in this case (and more generally for any neutral pseudoscalar meson) the hadron electromagnetic current reads

$$\mathcal{J}_\mu = \frac{F(t)}{m}\epsilon_{\mu\alpha\beta\gamma}k^\alpha e^\beta q^\gamma, \quad (8)$$

where e_β is the polarization four-vector of the real photon and $F(t)$ the electromagnetic form factor of the transition $\eta_c \to \gamma\gamma^*$, which has a nonzero imaginary part for $t \geq 4m_\pi^2$. Note that we have defined the form factor different from Lepage and Brodsky [20]. Inserting (8) into the definition of the tensor $W_{\mu\nu}$, (3), we obtain

$$W_1(t) = |F(t)|^2(1-x)^2\frac{m^2}{4}, \quad W_2(t) = 0,$$

i.e. $R_L(t) = 0$.

There are three different processes for studying the t dependence of F:

– The photoproduction of η_c in $\gamma e^- \to \eta_c e^-$ at space-like momentum transfers;
– The decay $\eta_c \to \gamma\ell^+\ell^-$, at time-like momentum transfers, $4m_\ell^2 \leq t \leq m^2$;
– The reaction $e^+e^- \to \eta_c\gamma$, at time-like momentum transfers, $t \geq m^2$.

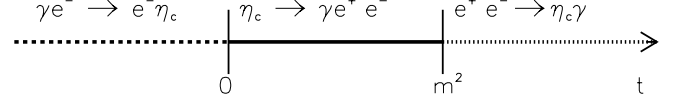**Fig. 3.** Scheme for the momentum transfer regions covered by the reactions $\gamma e^- \to \eta_c e^-$, $\eta_c \to \gamma e^+e^-$ and $e^+e^- \to \eta_c\gamma$

Therefore the simultaneous study of all these different processes allows one to determine the t dependence of the electromagnetic form factor $F(t)$ for any value of t (see Fig. 3). Moreover, all the possible observables in the above mentioned processes depend only on $|F(t)|^2$. This means that the phase of $F(t)$ in the time-like region cannot be measured. But the determination of this phase as a function of t is very important. For example, a dispersion relation for the form factor $F(t)$, considered as an analytical function of t, can be analyzed only if both the modulus and the phase of $F(t)$ are known. The problem of measuring the phase of complex form factors of hadrons in the time-like region is not yet solved. This is a common problem of the electromagnetic form factor of the charged pion, of the neutral and charged kaons, etc.

The determination of the transition form factor $F(t)$ is necessary for calculating the following observables:

– The Dalitz distribution in the variables x and y;
– The spectrum of the effective masses in $\eta_c \to \gamma\ell^+\ell^-$;
– The coefficient of internal conversion, i.e. the ratio of the decay width of $\eta_c \to \gamma\ell^+\ell^-$ to the decay width of $\eta_c \to \gamma\gamma$:

$$I = \frac{\Gamma(\eta_c \to \gamma\ell^+\ell^-)}{\Gamma_0} = \frac{2\alpha}{3\pi}\int_\delta^1 \mathrm{d}x \left|\frac{F(t)}{F(0)}\right|^2 \frac{(1-x)^3}{x}$$
$$\times \left(1 + 2\frac{m_\ell^2}{t}\right)\left(1 - \frac{4m_\ell^2}{t}\right)^{1/2}. \quad (9)$$

This coefficient derives its main contribution from small values of t, therefore it is sensitive to the mass of the lepton, in particular it is different in the decays $\eta_c \to \gamma e^+e^-$ and $\eta_c \to \gamma\mu^+\mu^-$.

3 Two models for $F(t)$

Now we consider the predictions of two models, the VDM and a pQCD-inspired non-relativistic model.

3.1 Vector dominance model

In the framework of VDM (Fig. 4), the t dependence of the electromagnetic form factor $F(t)$ can be written as

$$F(t) = \sum_V \frac{g_V}{t - m_V^2}, \quad g_V = m_V^2 f_V g(V \to \eta_c\gamma), \quad (10)$$

where we have summed over the contributions of the vector mesons. m_V is the vector meson mass, f_V is the constant of the $\gamma^* \to V$ transition and $g(V \to \eta_c\gamma)$ is the
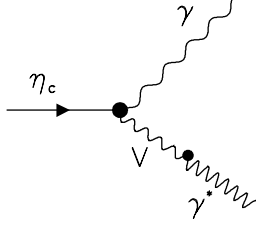
Fig. 4. Diagram for the process $\eta_c \to \gamma\gamma^*$ in the framework of VDM

Fig. 5. Diagrams for the process $c\bar{c} \to \gamma e^+e^-$

constant in the vertex $V \to \eta_c\gamma$. If $m_V \geq m$, the decay $V \to \eta_c\gamma$ is possible and the corresponding branching ratio allows one to determine the constant $g(V \to \eta_c\gamma)$, or, more exactly, as previously discussed, the modulus of this constant. Incidentally, notice that the constant f_V determines the decay width of $V \to \ell^+\ell^-$ (see Appendix).

The constants g_V satisfy two important conditions:

$$-\sum_V \frac{g_V}{m_V^2} = F(0), \tag{11}$$

$$\sum_V g_V = 0. \tag{12}$$

The first one is a relation between different g_V, through an evident normalization of $F(t)$. The constant $F(0)$ can be determined from the decay width of $\eta_c \to 2\gamma$, i.e., from

$$\Gamma_0 = \frac{\alpha}{4}|F(0)|^2 m, \tag{13}$$

which is experimentally known. Again, the value of the width Γ_0 does not fix the sign of $F(0)$, which introduces a two-fold ambiguity in (11).

The relation (12) results from a specific asymptotic behavior of $F(t)$: the structure of the current $\mathcal{J}_\mu$ shows that $F(t) \propto t^{-2}$ for large $|t|$. This differs from the standard $|t|^{-1}$ behavior of the elastic electromagnetic form factors of mesons; however, it agrees with the quark counting rule, and it results from the presence in (8) of an additional four-momentum, owing to the electromagnetic current of the decay considered.

The simplest model that can be suggested here for $F(t)$ must contain the contribution of two vector mesons. The problem is how to select the most important contributions.

Taking into account the $c\bar{c}$ nature of η_c, the J/ψ seems to be a good candidate, as confirmed by the value of the branching ratio $Br(J/\psi \to \eta_c\gamma) = (1.3 \pm 0.4)\%$. Similarly the branching ratio $\mathrm{Br}(\eta_c \to \rho\rho) = (2.6 \pm 0.9)\%$ exhibits the importance of the ρ contribution to the form factor $F(t)$. Therefore, in such a two-pole model we find the following two parameterizations:

$$F(t) = \pm |F(0)| \frac{m_\rho^2 m_{J/\psi}^2}{m_{J/\psi}^2 - m_\rho^2} \left(\frac{1}{t - m_\rho^2} - \frac{1}{t - m_{J/\psi}^2} \right), \tag{14}$$

which cannot be disentangled in the study of the above mentioned processes.

In order to avoid singularities at $t = m_V^2$, we have to substitute $m_V^2 \to m_V^2 - im_V\Gamma_V$, where Γ_V is the width of the V meson ($V = \rho, J/\psi$).
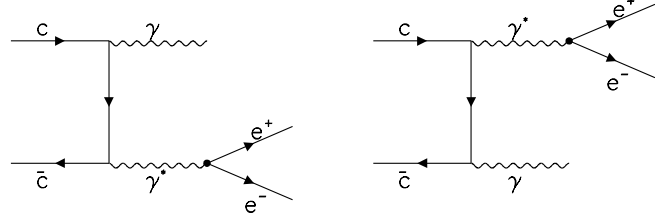
These parameterizations offer a numerical estimation of all the observables of the η_c photoproduction on electrons, $\gamma e^- \to \eta_c e^-$, and of the decay $\eta_c \to \gamma\ell^+\ell^-$, where $t \leq m^2$. In both processes the ρ contribution is particularly important at relatively small momentum transfers, $|t| \leq 1\,\mathrm{GeV}^2$. The J/ψ contribution plays an important role in the decay $\eta_c \to \gamma\ell^+\ell^-$, near the upper kinematical limit in the variable t, $t \to m^2$, where the J/ψ pole is very close to the physical region.

On the other hand this simplified parameterization is not convenient for the study of the annihilation process, $e^+e^- \to \eta_c\gamma$, to which the entire set of ψ resonances in principle could contribute. In this case formula (14) has to be generalized, taking into account, at least,

- the contribution of additional ψ resonances;
- the widths of such resonances, as illustrated above.

In the framework of VDM it is in principle possible to write a five-pole representation for $F(t)$, with the contributions of $\rho, J/\psi, \psi(3.77), \psi(4.04), \psi(4.41)$. The constants g_V of $\rho, J/\psi$ and $\psi(3.77)$ can be found using the existing experimental data about the radiative decays: $J/\psi \to \eta_c\gamma$, $\psi(3.77) \to \eta_c\gamma$, $\eta_c \to \rho\rho$, $\rho \to e^+e^-$, $J/\psi \to e^+e^-$ and $\psi(3.77) \to e^+e^-$. The details of the calculation are given in the Appendix. On the other hand, using relations (11) and (12), it is possible to find the constants g_V of the $\psi(4.04)$ and the $\psi(4.41)$ resonances. As two choices are possible for the sign of $F(0)$ and for the constants g_V for the first three resonances, $\rho, J/\psi$ and $\psi(3.77)$, one finds a $2^4 = 16$ fold parameterization for $F(t)$, which prevents any realistic estimation of the t behavior of the cross section for the process $e^+e^- \to \eta_c\gamma$. So our prediction of the various characteristics of the decay $\eta_c \to \gamma e^+e^-$ will be based on the simplified two-pole model for $F(t)$.

3.2 QCD-inspired model

Now we consider the process of $c\bar{c}$ quark annihilation into $\gamma\gamma^*$, which receives the contribution of the two diagrams illustrated in Fig. 5.

In the limit of $|\vec{p}| = 0$ (where $\vec{p}$ is the relative momentum of the $c\bar{c}$ system in the CMS), the $c\bar{c}$ annihilation in a singlet S state – which has the quantum numbers of the η_c meson, $J^P = 0^-$ – is described by a matrix element of the form

$$\mathcal{M}(c\bar{c}) \propto \vec{k} \cdot \vec{e} \times \vec{e}', \tag{15}$$

where $\vec{e}$ ($\vec{e}'$) is the polarization vector of the real (virtual) photon. The following relations hold: $\vec{e}' \cdot \vec{k} \neq 0$ and $\vec{e} \cdot \vec{k} = 0$.
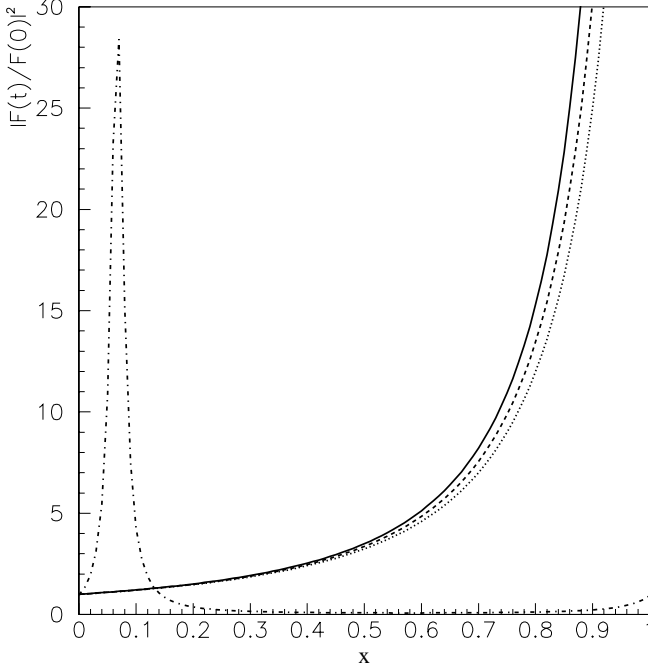
Fig. 6. x dependence (x $= t/m^2$) of $|F(t)/F(0)|^2$ in the VDM two-pole model (dashed-dotted line) and in the QCD-inspired model for different values of the transverse quark momentum: $2\langle\vec{k}_\perp^2\rangle/m^2 = 0.075$ (solid line), $2\langle\vec{k}_\perp^2\rangle/m^2 = 0.100$ (dashed line), and $2\langle\vec{k}_\perp^2\rangle/m^2 = 0.125$ (dotted line)

The result (15) can be found, without specific calculations, on the basis of symmetry properties relative to C and P transformations for the process $c\bar{c} \to \gamma\gamma^*$. Due to the positive C parity of the two final γs, the sum of the total spin S and the angular momentum ℓ of the $(c\bar{c})$ system must be even. But in the limit of $\vec{p} \to 0$ we have $\ell = 0$, therefore $S = 0$. In this limit the t dependence of the resulting matrix element for $c\bar{c} \to \gamma\gamma^*$ can be identified with the t dependence of the form factor $F(t)$ of the decay $\eta_c \to \gamma\gamma^*$. This dependence can be written in terms of the c quark propagators in the elementary process $c\bar{c} \to \gamma\gamma^*$, which are identical for both diagrams of Fig. 5. Therefore in the limit of $\vec{p} = 0$ we have

$$F(t) \propto \frac{1}{(p-k)^2 - m_c^2} = \frac{1}{k^2 - 2p\cdot k} \stackrel{\vec{p}=0}{=} \frac{1}{k^2 - 2m_c k_0}$$
$$= \frac{2}{k^2 - 4m_c^2},$$

which implies that $F(t) \propto |t|^{-1}$ for large $|t|$, unlike the VDM.

The parameter $4m_c^2$ which appears in the denominator should be identified with a physical quantity. The simplest possibility is to set $2m_c = m_{J/\psi}$, which corresponds to the J/ψ contribution to $F(t)$, appearing in the framework of VDM. A perturbative approach, on the basis of a factorization of short- and long-distance physics [20], as considered by Feldmann and Kroll [21,22] in the space-like region, results in

$$4m_c^2 \to m^2 + 2\langle\vec{k}_\perp^2\rangle, \qquad (16)$$

Table 1. Prediction for the coefficients of internal conversion in units of 10^{-2}. We have set $\Delta = 2\langle\vec{k}_\perp^2/m^2\rangle$

Model	$\eta_c \to \gamma e^+e^-$	$\eta_c \to \gamma\mu^+\mu^-$
VDM	3.99	2.39
QCD, $\Delta = 0.075$	2.22	0.61
QCD, $\Delta = 0.100$	2.22	0.60
QCD, $\Delta = 0.125$	2.21	0.69

where $\vec{k}_\perp$ denotes the transverse momentum of the c quark in the η_c meson [23]. This substitution may be regarded as a possible procedure for taking into account the transverse momentum of the quark inside the meson. On the other hand, note that our QCD-inspired consideration above is valid for any t, both space-like and time-like, except for a small neighborhood of $t = 4m_c^2$. Therefore, if we assume that the substitution (16) can be extended at least up to $t \leq m^2$, $F(t)$ turns out to be very sensitive to $\langle\vec{k}_\perp^2\rangle$ near the upper kinematical limit of the Dalitz plot, as results from Fig. 6.

In this approach it is also possible to find the corresponding form factors of the radiative decays $\chi_J \to \gamma\ell^+\ell^-$ of the P-wave charmonium states, $J = 0, 1$ and 2. To this end it is necessary to find the matrix element of the process $c\bar{c} \to \gamma\gamma^*$ (see Fig. 5). The spin structure of the χ decay matrix elements is

$$\xi_1^\dagger \sigma_2 \vec{\sigma} \cdot \vec{p}\xi_2 \vec{e}\cdot\vec{e}': \quad \chi_0 \to \gamma + \gamma^*,$$

$$\xi_1^\dagger \sigma_2 \vec{\sigma} \times \vec{p}\cdot \vec{e}\times\vec{k}\xi_2\vec{e}'\cdot\vec{k}: \quad \chi_1 \to \gamma + \gamma^*,$$

$$\xi_1^\dagger \sigma_2 \left(\sigma_a p_b + \sigma_b p_a - \frac{2}{3}\delta_{ab}\vec{\sigma}\cdot\vec{p}\right)\xi_2 \left\{\begin{array}{c} e_{1a} \quad e_{1b} \\ k_a k_b \quad \vec{e}\cdot\vec{e}' \\ e_{1a}k_b \quad \vec{e}'\cdot\vec{k} \end{array}\right\}:$$

$$\chi_2 \to \gamma + \gamma^*,$$

where $\xi_{1(2)}$ is the two-component spinor of the $\bar{c}$ (c) quark.

In the general case the decays of χ_0 and χ_1 are characterized by one form factor, whereas in the case of $\chi_2 \to \gamma\gamma^*$ three independent form factors are present. Notice also that the matrix element for $\chi_1 \to \gamma\gamma^*$ is proportional to $\vec{e}'\cdot\vec{k}$, which vanishes for real photons, in agreement with the Landau–Yang theorem.

As one can see from Fig. 6, the VDM and QCD-inspired model predict essentially different t dependences for the transition form factor $F(t)$ in the whole region $4m_\ell^2 \leq t \leq m^2$, where the decay $\eta_c \to \gamma\ell^+\ell^-$ occurs. The coefficients of internal conversion of the decays $\eta_c \to \gamma e^+e^-$ and $\eta_c \to \gamma\mu^+\mu^-$ are also strongly model dependent (see Table 1). These properties of $F(t)$ depend essentially on two factors: the role of the ρ meson in the VDM approach and the small mass difference between η_c and J/ψ.

Also in the space-like region the two models describe the form factor quite differently; in particular the QCD-inspired model predicts for $F(t)$ a much slower decrease at increasing $|t|$, in agreement with present data [10].

4 The reaction $e^+e^- \to \eta_c\gamma$

The differential cross section in the overall CMS reads

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 t}\frac{|\vec{q}|}{|\vec{k}|} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 t}\left(1 - \frac{m^2}{t}\right), \quad m_e = 0, \quad (17)$$

where $\vec{k}$ and $\vec{q}$ are the three-momenta of the colliding leptons and of the produced γ, and $t = (k_1 + k_2)^2$ is the square of the total energy of the colliding particles.

Taking into account (8), we get

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2}{32m^2}(1 + \cos^2\theta)\left(1 - \frac{m^2}{t}\right)^3 |F(t)|^2, \quad t \geq m^2. \tag{18}$$

The cross section for collinear kinematics (i.e. $\theta = 0$ or π) is non-vanishing, in agreement with helicity conservation. Indeed, in the limit of $m_e = 0$, owing to the properties of the electromagnetic current $\overline{u}\gamma_\mu u$, we have $\lambda_i = \lambda_{e^-} + \lambda_{e^+} = \pm 1$; on the other hand for a real γ we have $\lambda_f = \lambda_\gamma = \pm 1$.

In any process of the type $e^+e^- \to \gamma^* \to h_1\overline{h_2}$, where h_1 and h_2 are hadrons or hadronic states, the one-photon mechanism is characterized by a definite angular dependence:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = A(t) + B(t)\cos^2\theta,$$

where θ is the production angle of h_1 in the overall CMS and $A(t)$ and $B(t)$ are two structure functions which are quadratic combinations of the electromagnetic form factors of the transition $\gamma^* \to h_1\overline{h_2}$. In the case considered we have only one form factor for the vertex $\gamma^* \to \eta_c\gamma$; therefore $B(t)/A(t) = 1$, as results from the angular dependence (18).

The threshold behavior of the cross section is proportional to $\left(1 - m^2/t\right)^3$ and results from the magnetic dipole radiation in the transition $\gamma^* \to \eta_c\gamma$.

The factorization of the t and $\cos\theta$ dependences in the differential cross section of $e^+e^- \to \eta_c\gamma$, (18), is due to the specific structure of the matrix element and to the choice of the CMS as a reference frame. Therefore the t dependence is the same for any production angle and coincides with the t dependence of the total cross section:

$$\sigma = \frac{\pi}{6m^2}\alpha^2\left(1 - \frac{m^2}{t}\right)^3 |F(t)|^2. \tag{19}$$

Equations (18) and (19) can be rewritten as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha}{2m^3}\Gamma_0\left(1 - \frac{m^2}{t}\right)^3\left|\frac{F(t)}{F(0)}\right|^2(1 + \cos^2\theta)$$

$$= \frac{(1 + \cos^2\theta)}{16\pi}3\sigma,$$

$$\sigma = \frac{4}{3}\frac{\alpha\pi}{m^3}\Gamma_0\left(1 - \frac{m^2}{t}\right)^3\left|\frac{F(t)}{F(0)}\right|^2.$$

In order to predict the t dependence of the total and differential cross sections, it is necessary to know the form

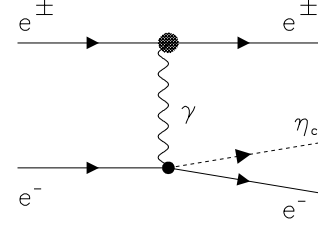**Fig. 7.** Feynman diagram for $e^\pm e^- \to e^\pm e^- \eta_c$

factor $F(t)$ in the region $t \geq m^2$, and especially where the contribution of all the ψ resonances is important. From our previous discussion it turns out that the VDM can be applied here.

5 The process $\gamma e^- \to \eta_c e^-$.

The direct study of the reaction $\gamma e^- \to \eta_c e^-$ is experimentally difficult due to the large threshold, $E_{\gamma,\mathrm{thr}} \simeq m^2/2m_e \simeq 9\,\mathrm{TeV}$. Therefore this process can be studied through the 2γ mechanism in the reaction $e^\pm e^- \to e^\pm e^-\eta_c$ (see Figs. 7 and 1) with a quasi-real photon produced in the $e^\pm \to e^\pm\gamma$ vertex. In this case, for the process $\gamma e^- \to \eta_c e^-$, it is necessary to consider a particular reference frame with colliding γ and e^-. Let us re-write the cross section of $\gamma e^- \to \eta_c e^-$ in terms of the Mandelstam variables s, t and u, which can be defined in the standard form, according to the following notation of the particles four-momenta:

$$\gamma(q) + e^-(k_1) \to e^-(k_2) + \eta_c(p),$$

$$s = (k_1 + q)^2,$$

$$u = (k_2 - q)^2,$$

$$t = (k_1 - k_2)^2,$$

where q, p and k_1 (k_2) are the four-momenta of the photon, of η_c and of the initial (final) electron. In the overall CMS the differential cross section can be written as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_\eta}(\gamma e^- \to \eta_c e^-) = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 s}\left(1 - \frac{m^2}{s}\right).$$

In the one-photon exchange approximation the expression of $|\overline{\mathcal{M}}|^2$ can be derived from (5), by exploiting the crossing symmetry. In the direct channel we have

$$\overline{|\mathcal{M}|^2} = \frac{e^4|F(t)|^2}{m^2 t}(1 + \cos^2\theta)(t - m^2)^2, \tag{20}$$

where t is the overall CMS energy squared.

Taking into account the relation

$$1 - \cos^2\theta = \frac{4su}{(t - m^2)^2},$$

the invariant differential cross section for $\gamma e^- \to e^- \eta_c$ is found to be

$$\frac{d\sigma}{dt} = \frac{\pi\alpha^2}{4s^2 m^2 |t|}[(t-m^2)^2 - 2us]|F(t)|^2$$

$$= 4\pi \frac{\alpha}{m^3} \Gamma_0 \left|\frac{F(t)}{F(0)}\right|^2 \frac{(t-m^2)^2 - 2us}{s^2|t|}. \quad (21)$$

Since the four-momentum transfer in this process is space-like, the reaction considered is especially sensitive to the ρ meson contribution, which is the lightest vector meson – in particular for $|t| \leq 1\,\text{GeV}^2$.

In the framework of the one-photon mechanism a definite prediction for the polarization observables can be derived. Independently of the kinematical conditions, the absorption asymmetry of a linearly polarized photon, i.e.,

$$\Sigma = \frac{\sigma_\perp - \sigma_\parallel}{\sigma_\perp + \sigma_\parallel},$$

where $\sigma_\perp$ ($\sigma_\parallel$) is the differential cross section with linearly polarized photons, whose polarizations are orthogonal (parallel) to the reaction plane, is equal to 1. This result, which implies $\sigma_\parallel = 0$, follows from parity conservation.

This prediction can be tested using a linearly polarized photon beam and detecting the final electron. The electron polarization here does not carry any additional information, owing to the presence of a single electromagnetic form factor. However, for two-spin polarization observables non-trivial effects can be produced by the collisions of circularly polarized photons with longitudinally polarized electrons.

6 Conclusions

We have considered some characteristics of the radiative decay $\eta_c \to \gamma\ell^+\ell^-$, such as the Dalitz distribution with respect to the energies of the two final leptons, the effective mass spectrum of the lepton pair and the coefficient of internal conversion, i.e. the ratio $\Gamma(\eta_c \to \gamma\ell^+\ell^-)/\Gamma(\eta_c \to 2\gamma)$. We have used two different models for the electromagnetic form factor $F(t)$. The VDM allows one to find a relatively simple two-pole representation for the form factor $F(t)$, with a definite normalization at $t = 0$ and an asymptotic behavior in t^{-2}, in agreement with the quark counting rule. The VDM parameterization contains the contribution of the ρ meson (which is important owing to the large width of the decay $\eta_c \to \rho^0\rho^0$) and the J/ψ contribution. This two-pole representation can be used for analyzing the decay $\eta_c \to \gamma\ell^+\ell^-$, which corresponds to time-like t, and the photoproduction process $\gamma e^- \to \eta_c e^-$, where t is space-like. However, in the calculation of the energy dependence of the differential and total cross section of the process $e^+e^- \to \eta_c\gamma$ the form factor $F(t)$ could be sensitive to the whole set of ψ resonances. A suitable parameterization of the form factor, in terms of the contribution of the vector mesons ρ, J/ψ, $\psi(3.77)$, $\psi(4.04)$ and $\psi(4.41)$, can be found in the framework of VDM, but
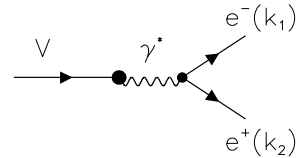

Fig. 8. Diagram of the decay $V \to e^+e^-$, with notation of particle four-momenta, in the framework of the one-photon exchange mechanism
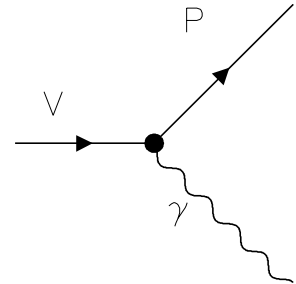

Fig. 9. Diagram of the decay $V \to P\gamma$

with a $2^4 = 16$ fold ambiguity in the definition of the signs of the various constants. The QCD-inspired model for the form factor $F(t)$ on the basis of the process $c\bar{c} \to \gamma\ell^+\ell^-$ (with two-pole Feynman diagrams) results in a parameterization which is very similar to the J/ψ contribution alone. This model predicts an asymptotic behavior of the type $|t|^{-1}$ for large $|t|$, in contrast with the VDM.

Appendix

Here we consider the decays $\eta_c \to \rho^0\rho^0$, $V \to \eta_c\gamma$ and $V \to \ell^+\ell^-$, whose decay constants are necessary for the calculation of the form factor $F(t)$ in VDM.

The matrix element for the process $\eta_c \to \rho^0\rho^0$ can be written as

$$\mathcal{M} = \frac{g(\eta_c \to \rho^0\rho^0)}{m} \epsilon^{\mu\nu\alpha\beta} U_{1\mu} U_{2\nu} p_{1\alpha} p_{2\beta},$$

where $U_1(U_2)$ and $p_1(p_2)$ are the polarization four-vectors and four-momenta of the two produced particles. Summing over the final polarizations, the partial decay width reads

$$\Gamma(\eta_c \to \rho^0\rho^0) = \frac{m}{64\pi}|g(\eta_c \to \rho^0\rho^0)|^2 \left(1 - \frac{4m_\rho^2}{m^2}\right)^{3/2},$$

the constant $g(\eta_c \to \rho^0\rho^0)$ being dimensionless in the present normalization.

The matrix element of the decay $V \to \ell^+\ell^-$ (see Fig. 8, which illustrates the notation of the particle four-momenta), can be written as

$$\mathcal{M} = e^2 f_V \bar{u}(k_1)\gamma_\mu u(k_2) U^\mu,$$

where U_μ is the polarization four-vector of the V meson. Averaging over U_μ and summing over the final lepton polarizations, we find, in the limit of zero leptonic mass,

$$\Gamma(V \to \ell^+\ell^-) = \alpha^2 f_V^2 \frac{4\pi}{3} m_V.$$

The matrix element of the decay $V \to P\gamma$, where P is a pseudoscalar meson (Fig. 9), can be written as

$$\mathcal{M} = e \frac{g(V \to P\gamma)}{m_V} \epsilon^{\mu\nu\alpha\beta} U_\mu e_\nu q_\alpha k_\beta.$$

Summing over the γ polarizations and averaging over the V polarizations yields

$$\Gamma(V \to P\gamma) = \frac{\alpha g^2(V \to P\gamma)}{24} m_V \left(1 - \frac{m^2}{m_V^2}\right)^3.$$

Finally, g_V is connected to $g(V \to P\gamma)$ and f_V through the relation

$$g_V = e f_V g(V \to P\gamma) m_V^2.$$

We stress that this relation does not fix the sign of the constant g_V.

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